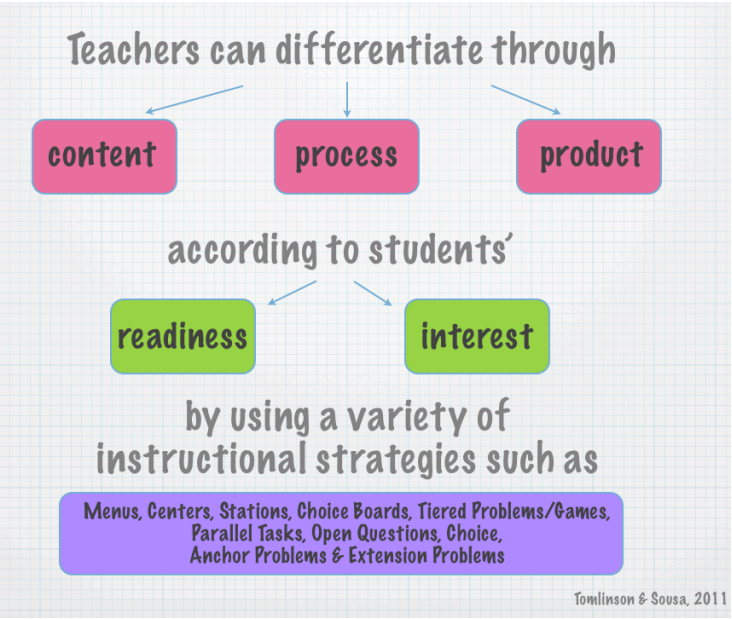
Differentiation in the Elementary Mathematics Classroom

VVCTM Fall Conference, October 7, 2014

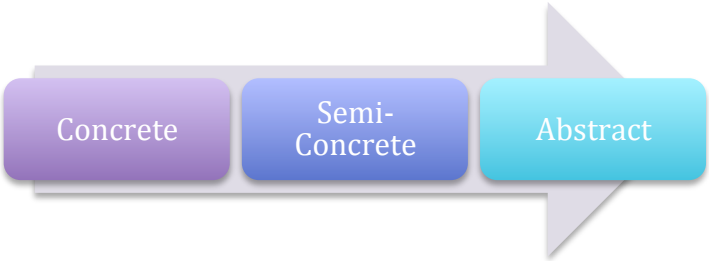
Kateri Thunder, Ph.D.



Process Learning Progressions

Levels of Representation

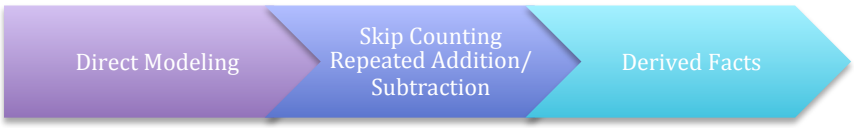
(Van de Walle et al., 2014)



Levels of Problem Solving

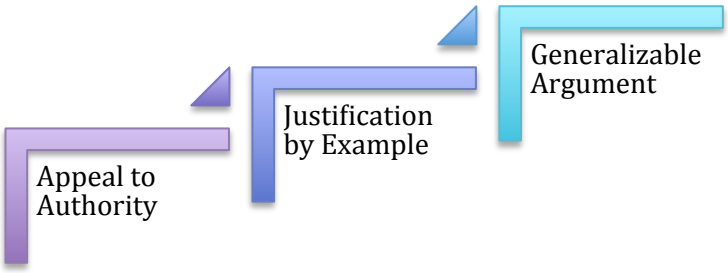
(Carpenter et al., 1999)

Addition and Subtraction

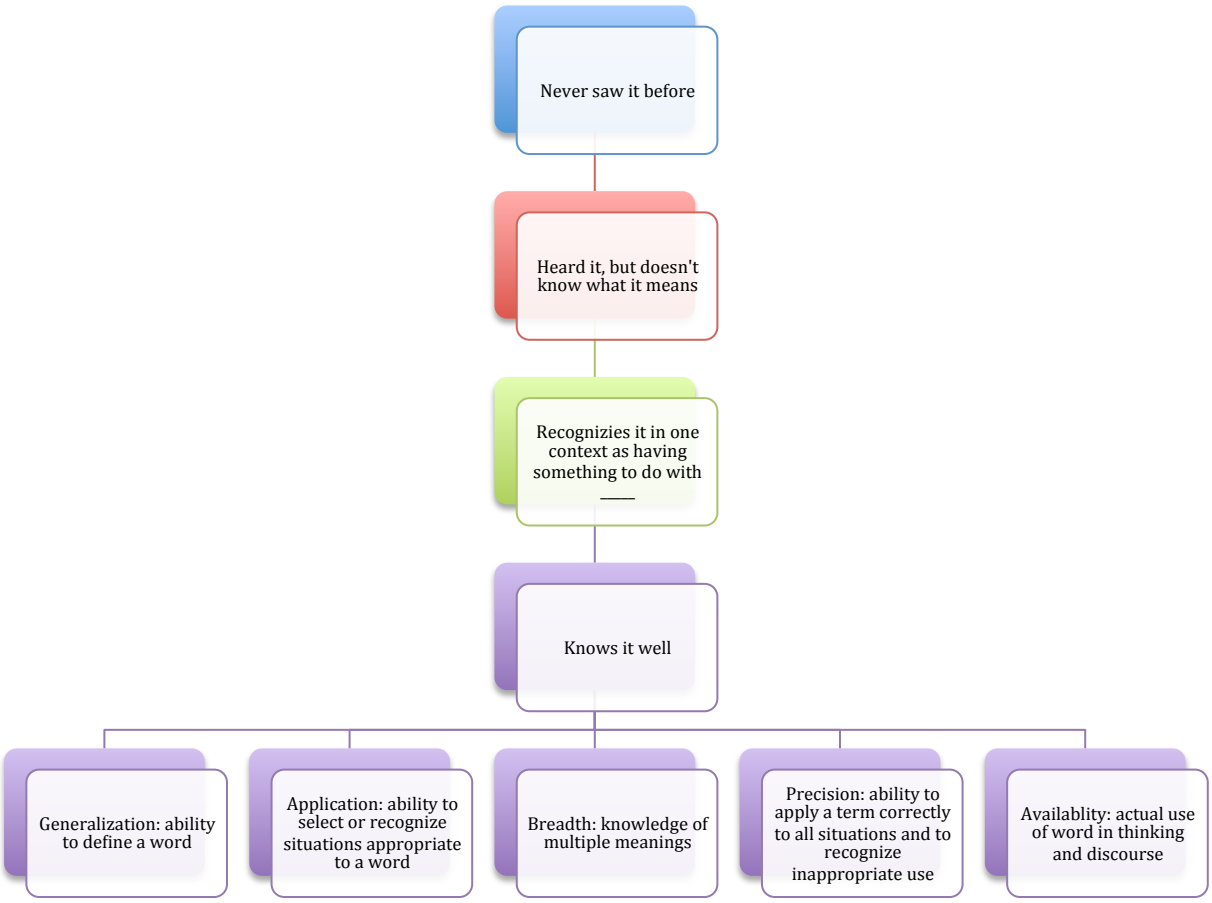


Multiplication and Division

Levels of Justification
(Carpenter et al., 2003)



Stages of Word Knowledge
(Beck et al., 2002)

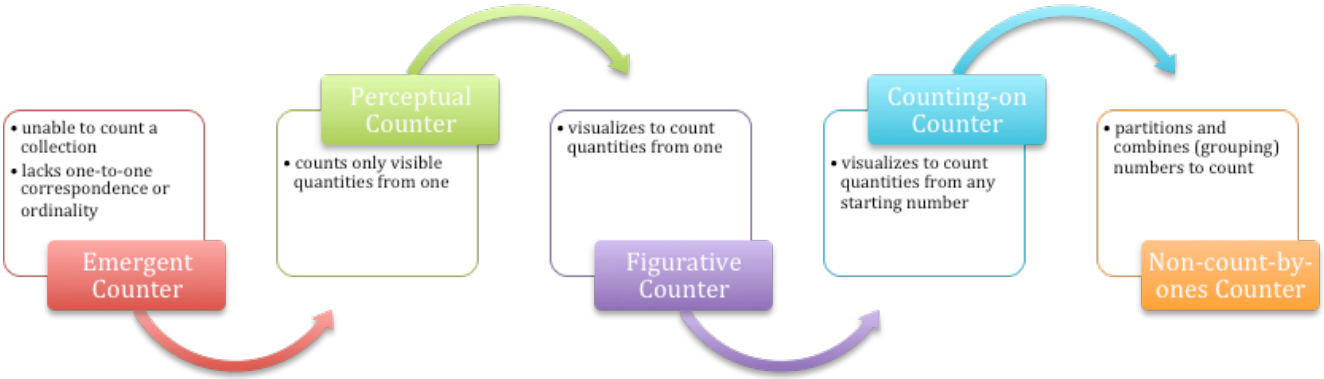


Content Learning Progressions (a few)

Counting

(Van de Walle et al., 2014)

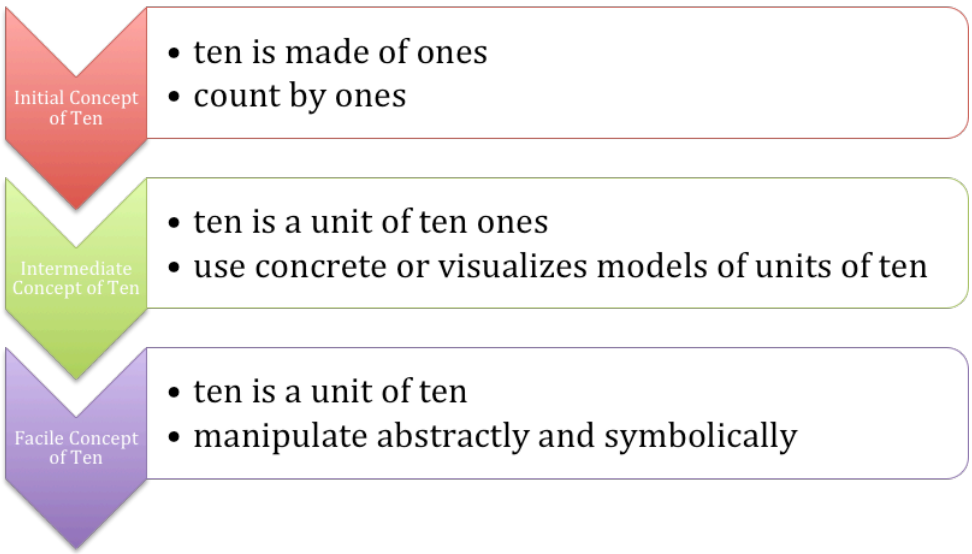
Counting tells how many.



Concept of Ten

(Van de Walle et al., 2014)

Quantities can be regrouped into combinations of multiples of ten.



Addition and Subtraction Problem Types

(Common Core State Standards Learning Progression, 2009)

Addition is combining. Subtraction is taking away or finding the difference.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$

	Total Unknown	Both Addends Unknown[1]	Addend Unknown[2]
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$

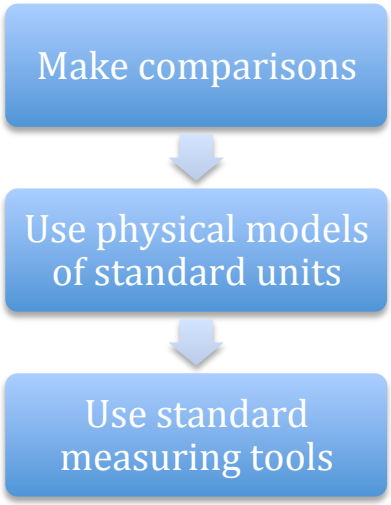
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Dark Gray → Light Gray → White

van Hiele Levels of Geometric Thinking
(Van de Walle et al., 2014)

Concepts	**Geometric Shapes and Properties**
Big Idea or Understanding	Geometric shapes are named and categorized based on their properties.
Progression of Strategies and Representations/Tools	Van Hiele: Pre-Recognition *Notice only one characteristic of shapes*
	Van Hiele: Visualization CCSS: Visual *Recognize/Identify classes/groups of shapes*
	Van Hiele: Analysis CCSS: Descriptive *Describe properties of shapes*
	Van Hiele: Informal Deduction CCSS: Analytic *Categorize shapes using if-then to show relationships between properties*
	Van Hiele: Deduction & Rigor CCSS: Abstract *Use deductive reasoning about relationships; Build on and connect axioms/definitions*

Measurement
(Van de Walle et al., 2014)



Samples of Differentiation:

Differentiating the Content by Interest

Anchor Problem and Extension Problems:

Caps For Sale

First he had on his own checked cap,
then a bunch of gray caps,
then a bunch of brown caps,
then a bunch of blue caps,
and on the very top a bunch of red caps.

The peddler has seventeen caps. How many of each color cap does he have?

Extension Problem 1:
The peddler begins his day with seventeen caps. When the peddler returns home that night, he has nine caps. How many caps did he sell?

Extension Problem 2:
A peddler in the neighboring town also sells caps. The neighboring peddler has eleven fewer caps than our peddler. Our peddler has seventeen caps. How many caps does the neighboring peddler have?

Number & Operations with Measurement:
Three-in-a-Row Board
Directions: Choose one task from each column.
Cross out each task as you complete it to make three in a row straight across or diagonal.

Money	***Time: Clocks***	***Time: Calendars***
Collect $1 Roll a dice. Collect the number of pennies on your dice.	***Collecting Time Data*** Estimate about how many seconds each activity will take you to do. Measure the actual time using a stopwatch. Record your data.	**Patterns in Calendars** Create a calendar for the year ____ using the blank templates.
Set: Dollars Make a 3x4 grid with12 face-up coin cards. Players look for pairs of cards to make one dollar.	***My Schedule*** Create your daily schedule for a typical Tuesday. Create analog and digital clocks for each time.	***Famous People and Places*** Create a timeline. Place the following dates:
Currency Around the World Explore the coins from around the world. Design your own currency.	***Times Around the World*** Estimate about what time you think it is in each city right now. Research to find out the actual time in each city. What is the time difference between Harrisonburg and each city?	**How Much is a Million?** How long is one million seconds? • Describe an equivalent period of time using large units that are easy to comprehend. Explain your strategy. • When is one million seconds from now?

Area Measurement Dinner Menu
A Differentiation Strategy

Appetizer (Everyone) 

- Which rectangle is the biggest?
  - Anticipate as many strategies as you can for ways students might sequence the rectangles by area.

Main Dish (Choose 1) 

How can you find the area of any _____?

1. Anticipate multiple strategies students could use to find the area of any parallelogram. Develop a formula for the area of any parallelogram.
2. Anticipate multiple strategies students could use to find the area of any triangle. Develop a formula for the area of any triangle.
3. Anticipate multiple strategies students could use to find the area of any trapezoid. Develop a formula for the area of any trapezoid.

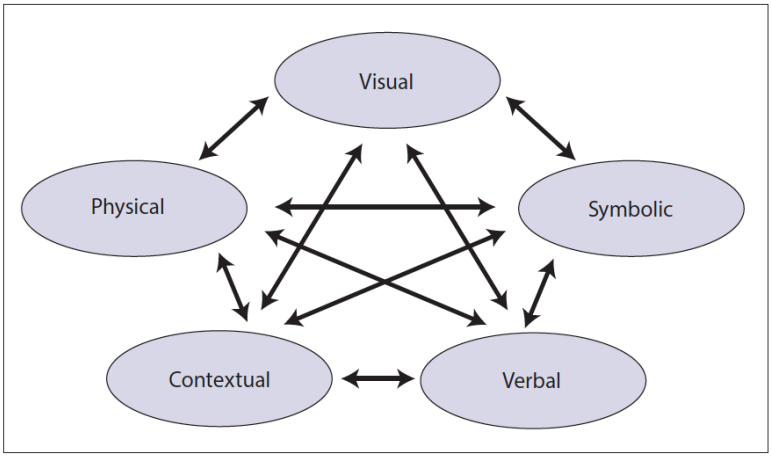
Side Dishes (Choose 1) 

How can you use what you know about decomposing and composing geometric shapes to find the area of unfamiliar or new shapes?

A. There is a mass of trash in the Pacific Ocean known as a garbage patch. The area of the Great Pacific Ocean garbage patch is the same as the area of the state of Texas. Find this area.
   - Here is a website with more information about the garbage page: http://www.greenbang.com/10-things-you-should-know-about-the-great-pacific-garbage-patch_21885.html

Differentiating the Product by Interest

Represent the pattern using your choice of representation:



Our list of genres:

- Catalog
- Bumper sticker
- Blogs
- Memes
- Texts
- Twitter
- Facebook
- Menus
- Road signs
- Ingredient labels
- Tattoos
- Plaques
- Grave markers
- Logos
- CNN Ticker
- Top ten lists
- Jokes
- LP Covers/CDs
- Comment sections
- Prescription
- World records
- Baseball or trading cards
- Aerial banners
- Calendars

Shapes and Their Properties
Project-Based Learning

Essential Question: How are shapes related to each other? How are shapes categorized and named?

Choose one project to complete for 2D and 3D shapes.
After creating your project, write a description of your process and answer the essential question using your project.

Cross-curricular Connection	*Project*
Science	Shape Classification Key (or Dichotomous Key)
Social Studies	Shape Family Tree
P.E.	Shape Trading Cards (Online Interactive: http://www.readwritethink.org/classroom-resources/student-interactives/trading-card-creator-30056.html)
Visual Arts	Shape Concept Map (digital - Coggle, Kidspiration – or physical)

Differentiating the Content by Readiness

Open Question:

The Box Factory
Design Brief

Background:

We have learned about spatial structuring, decomposing shapes, and measuring length and area. These concepts and skills are all closely related to volume.

Challenge:

A box factory hires you to design boxes of different sizes and shapes. Each box must hold 24 items. They already make a box with two rows by six columns of chocolates. That box has two layers.

You must determine the number of different arrangements of rows and columns and layers possible to hold 24 items. You must also prepare a presentation of the box design options and defend your suggestions.

Adjusting Numbers:

Ashley checks the movie times at the theatre. Her favorite movie is showing _____ .
She looks at the clock and it is 8:42am. How much time does she have till the movie begins?

a) today at 11:45am b) tonight at 6:10pm c) tomorrow at 12:35pm

Tiered Problems/Games or Parallel Tasks:

Tiered Addition/Subtraction Games:
Compatible Numbers

Essential Question: *What are compatible numbers? How can they help you add and subtract fluently?*

Tier 1	**Collecting Dollars**
Tier 2	**Close to 100**
Tier 3	**Close to 1,000**

For your game, what is a strategy that will always work? Why does it work?
How did you use compatible numbers in your game?

Dot Card Compare Station

Differentiation Strategy: Parallel Tasks

Choose one version of the game to play.

Compare	***Double Compare***	***Difference***
• Divide the deck of dot cards in half with your partner. • At the same time, each player turns up one card. • The player with the greater number of dots wins and takes both cards. • The game is over when one player has all of the dot cards and wins.	• Divide the deck of dot cards in half with your partner. • At the same time, each player turns up two cards. • The player with the greater total number of dots wins and takes all four cards. • The game is over when one player has all of the dot cards and wins.	• Divide the deck of dot cards in half with your partner. Also prepare a pile of 30 to 40 counters. • At the same time, each player turns up one card. • The player with the greater number of dots wins as many counters from the pile as the difference between the two cards. The players keep their cards. • The game is over when the counter pile runs out. The player with the most counters wins the game.

Definitions:

Differentiation: (Tomlinson & Sousa, 2011)

- **Content:** changing *what you teach*, still making sure everything you teach is under the same essential UKDs or learning objectives
- **Interest**: feeling/emotion that causes each student to focus on something because it matters to him/her always (personal interest inherent to the person) or just in the certain conditions that sparked the interest initially (situational interest based on choice within the context)
- **Process:** *how* students solve problems to meet the learning objectives: who they work with, what materials/resources they use, and/or what strategies they use
- **Product:** changing *what result* students create in order to show their understanding of the content and the process; this occurs at critical points in the unit and at the end
- **Readiness:** what each student knows and where on the continuum of mastery each student lies in regard to the understandings, knowledge, and skills (UKDs) needed prior to the unit and the UKDs in the unit; use preassessments and formative assessments to determine readiness levels; also consider misconceptions

Mathematical Proficiency: (National Research Council, 2001)

- **Adaptive Reasoning:** capacity for logical thought, reflection, explanation, and justification
- **Conceptual Understanding:** comprehension of mathematical concepts, operations, and relations
- **Procedural Fluency:** skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Productive Disposition:** habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy
- **Strategic Competence:** ability to formulate, represent, and solve mathematics problems

Learning Progression/Trajectory: a sequence of levels of understanding, knowledge, and skills within mathematical content and process standards. Each progression is made up of a big idea, a progression of strategies and mathematical models (representations and/or tools)

Resources:

Differentiation

- *Good Questions: Great Ways to Differentiate Mathematics Instruction* by Marion Small
- *Differentiation and the Brain: How Neuroscience Supports the Learner-Friendly Classroom* by David A. Sousa & Carol Ann Tomlinson
- *The Differentiated Classroom: Responding to the Needs of All Learners* by Carol Ann Tomlinson
- *Differentiated Instructional Strategies in Practice* by Gail Gregory
- *Differentiated Instructional Strategies: One Size Doesn't Fit All* by Carol Chapman
- *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades PreK-2, 3-5, and 6-8* by John A. Van de Walle, Karen S. Karp, LouAnn H. Lovin, and Jennifer M. Bay-Williams
- *English Language Learners in the Mathematics Classroom* by Debra Coggins, Drew Kravin, Grace Davila Coates, and Maria Dreux Carroll
- *Teaching Inclusive Mathematics to Special Learners, K-6* by Julie A. Sliva
- *5 Practices for Orchestrating Productive Mathematics Discussions* by M. Smith and M. Stein

Learning Progressions

- *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School* by Thomas P. Carpenter, Megan Loef Franke, and Linda Levi (Levels of Justification)
- *Bringing Words to Life: Robust Vocabulary Instruction* by Isabel L. Beck, Margaret G. McKeown, and Linda Kucan (Stages of Word Knowledge)
- *Children's Mathematics: Cognitively Guided Instruction* by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, and Susan B. Empson (Levels of Problem Solving)
- Content Learning Progressions for the Common Core State Standards (CCSS) in Mathematics. *These texts are available digitally for free:* http://ime.math.arizona.edu/progressions/
    - *Draft* K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking
    - *Draft* K-5 Progression on Number and Operations in Base Ten
    - *Draft* 3-5 Progression on Number and Operations – Fractions
    - *Draft* K-5 Progression on Geometry
    - *Draft* K-5 Progression on Measurement and Data (measurement part)
    - *Draft* K-5 Progression on Measurement and Data (data part)
    - *Draft* 6-8 Progression on Statistics and Probability
- *Extending Children's Mathematics: Fractions and Decimals – Innovations in Cognitively Guided Instruction* by Susan B. Empson and Linda Levi
- *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades PreK-2, 3-5, and 6-8* by John A. Van de Walle, Karen S. Karp, LouAnn H. Lovin, and Jennifer M. Bay-Williams